Letters

Comments on "Some Important Properties of Waveguide Junction Generalized Scattering Matrices in the Context of the Mode Matching Technique"

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The goal of this paper is to provide another reason in addition to those given in Section II of the above paper[1] in response to the rhetorical question, "Which mode-matching equations are linearly independent?" When the mode-matching method [1] is used to obtain the generalized scattering matrix of a discontinuity between two waveguides of different cross sections, it is essential to make an adequate choice of the eigenfunctions, which will be used to test the continuity equations of the tangential components of the electromagnetic field in the plane of the discontinuity. It is known that the testing eigenmodes are chosen as being those of the larger guide for enforcing the electric-field continuity and as those of the smaller guide for enforcing the magnetic-field continuity [2]. This means that the electric-field test eigenmodes must be chosen from the smaller guide and the magnetic-field test eigenmodes must be chosen from the larger guide. In the above paper, the authors prove this, first qualitatively and then rigorously. We would like to add the following reason that lends further support to the point-of-view presented in the above paper.

As in the above paper, suppose that $\mathrm{s1}$ is the cross section of the smaller guide and that $\mathrm{s2}$ is the cross section of the larger guide (see Fig. 1 in the above paper). The continuity equations for the tangential fields across the cross-section discontinuity in $z = 0$ are

$$\vec{E}_t^{(2)} = \vec{E}_t^{(1)}, \qquad \text{on } \mathrm{s1} \tag{1}$$

$$\vec{E}_t^{(2)} = 0, \qquad \text{on } (\mathrm{s2} - \mathrm{s1}) \tag{2}$$

$$\vec{H}_t^{(2)} = \vec{H}_t^{(1)}, \qquad \text{on } \mathrm{s1}. \tag{3}$$

However, there is one further condition on the tangential magnetic field, which is not normally written, but must be fulfilled at the discontinuity plane $z = 0$

$$\vec{n} \times \vec{H}^{(2)} = \vec{J}_{(s)}, \qquad \text{on } (\mathrm{s2} - \mathrm{s1}) \tag{4}$$

where $\vec{n}$ is the normal (in this case, $\vec{a}_z$) to the perfect conductor at the plane $z = 0$. It should be pointed out that this equation is not explicitly taken into account in any mode-matching formulation, although it is a boundary-condition independent of those given by (1)–(3). The reason for this is that boundary condition (4) involves added difficulty since it introduces two unknowns: the magnetic field $\vec{H}$ and the surface current density $\vec{J}_s$.

The mode-matching method involves every one of the tangential components of the electromagnetic field and is expressed as a series expansion in terms of N (guide #1) and M (guide #2) eigenmodes in accordance with (2) and (3) of the above paper. Moreover, for the solution of the problem to be correct, (1) and (2), which can easily be combined into a single equation, must be tested with the M tangential magnetic enigenmodes corresponding to the larger guide. Equation (3) is then tested with the N tangential electric eigenmodes corresponding to the smaller guide. This is the normal procedure. However, if (4) is taken into account, then (3) and (4) can be combined into a single equation, which is then tested with the N tangential electric eigenmodes corresponding to the smaller guide. The result is the same as if only (3) is tested because the eingenmodes corresponding to the electric field are valid on $\mathrm{s1}$ and (4) on $\mathrm{s2} - \mathrm{s1}$ (see Fig. 1 of the above paper); an identity equation would then be obtained from boundary condition (4). In other words, boundary condition (4) is not explicitly involved because it is converted in an identity when applying the method of moments to solve the system of equations from which the generalized scattering matrix is obtained. This is clearly the correct way to proceed because if (4) were to be included in the process, it would mean introducing a set of equations involving the surface current density $\vec{J}_s$, which is unknown, since the transverse magnetic field is also unknown. That is to say, an additional unknown would be introduced without adding more equations, and the resulting system would be impossible to solve.

On the other hand, if the combination of (3) and (4) is tested with the M tangential electric eingenmodes corresponding to the larger guide, the result is a set of equations in which we once again have the undesirable situation described in the above paragraph: the current density does not "disappear" from the formulation and, therefore, the system of equations does not have any correct solution. If only (3) is tested, the result would not be correct because it would not include the boundary condition for the surface current density since, in this case, it does not disappear as it does in the correct situation mentioned above.

References

[1] T. S. Chu and T. Itoh, "Generalized scattering matrix method for analysis of cascaded and offset microstrip step discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 280–284, Feb. 1986.

[2] H. Patzelt and F. Arndt, "Double-plane steps in rectangular waveguides and their application to transformers, irises and filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 771–776, May 1982.

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[1] G. V. Eleftheriades, A. S. Omar, L. P. B. Katehi, and G. M. Rebeiz, *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 10, pp. 1896–1903, Oct. 1994.

Authors' Reply

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In the mode-matching method, N modes of the waveguide with the smaller cross section S_1 are matched to M modes $(M > N)$ of the waveguide with the larger cross section S_2. In order to keep

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TABLE I
POSSIBLE TESTING EQUATIONS

Continuity of:	Testing Modes	Testing Cross Section	Describing Equation
$\mathbf{E}_{tangential}$	M modes of guide #2	S_2	Equation (6) of [1]
$\mathbf{E}_{tangential}$	M modes of guide #2	S_1	Equation (6) of [1], as the tangential electric field vanishes on $(S_2 - S_1)$
$\mathbf{E}_{tangential}$	N modes of guide #1	S_2	Irrelevant, as these modes are not defined on $(S_2 - S_1)$
$\mathbf{E}_{tangential}$	N modes of guide #1	S_1	Equation (7) of [1]
$\mathbf{H}_{tangential}$	M modes of guide #2	S_2	***The possibility we described above***
$\mathbf{H}_{tangential}$	M modes of guide #2	S_1	Equation (15) of [1]
$\mathbf{H}_{tangential}$	N modes of guide #1	S_2	Irrelevant, as these modes are not defined on $(S_2 - S_1)$
$\mathbf{H}_{tangential}$	N modes of guide #1	S_1	Equation (14) of [1]

the same spatial resolution everywhere, the cutoff frequencies of the highest order modes in either guides have to be more or less equal. This should apply to the short-circuited area $(S_2 - S_1)$ as well. In other words, if the surface magnetic current $\mathbf{J}_s = \mathbf{n} \times \mathbf{H}_t$ is to be expanded in terms of the eigenmodes of a virtual waveguide with the cross section $(S_2 - S_1)$, a number of modes $(M - N)$ should be used.

The standard mode-matching technique tests the continuity of the tangential electric field by the M modes of the larger guide and the continuity of the tangential magnetic field by the N modes of the smaller guide. This gives rise to $(N + M)$ linear equations relating the $(2N + 2M)$ modal expansion coefficients (N modal voltages and N modal currents in guide #1 in addition to M modal voltages and M modal currents in guide #2).

The possibility of testing both continuities by the M modes of the larger guide addressed in the comments of Solano *et al.* is, in fact, an extension of the standard mode-matching technique described above. It would result in $2M$ linear equations instead of the standard $(N + M)$ ones. The additional $(M - N)$ equations can, however, be used to determine $\mathbf{J}_s$, as $\mathbf{J}_s$ can be expanded in terms of the first $(M - N)$ modal magnetic fields of the virtual waveguide with the cross section $(S_2 - S_1)$, as mentioned above.

We do not, however, agree with the last sentence of the comments of Solano *et al.*, i.e., that testing the continuity of the tangential magnetic field by the M modes of the larger guide would result in a wrong formulation if the testing cross section is the smaller one (S_1) [(15) in the above paper[1]], as we have proven the correctness of this equation in the above paper. In order to clarify the whole situation, let us summarize all possible testing equations (a total number of eight possibilities), as shown in Table I.

[1]G. V. Eleftheriades, A. S. Omar, L. P. B. Katehi, and G. M. Rebeiz, *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 10, pp. 1896–1903, Oct. 1994.

Comments on "Relationship Between Group Delay and Stored Energy in Microwave Filters"

Christoph Ernst and Vasil Postoyalko

In the above paper,[1] it is shown that the time-averaged stored energy in a passive lossless reciprocal symmetrical or antimetrical two-port is proportional to the group delay. This is expressed in (29) in the above paper, i.e.,

$$W_{\mathrm{av,tot}} = -|a_1|^2 \frac{d\phi_{21}}{d\omega}. \tag{1}$$

In a private communication, Cuthbert pointed out that "*[...] it appears that the same results you obtained were previously reported in the book by Paul Penfield, Robert Spence, and Simon Duinker* [1]. *The relevant pages are 64 to 67, especially Section 5.17 Group Delay and Stored Energy. It is interesting to note Penfield et al. attributing some particular results to Dicke* [2], *Kishi and Nakazawa* [3], *and Carlin* [4], *which you did not reference*." Reference [1, eq. (5.82)],

$$\frac{d\theta_{12}}{d\omega} + \frac{d\theta_{21}}{d\omega} = \frac{(W_e + W_m)_1 + (W_e + W_m)_2}{P} \tag{2}$$

which is attributed to Carlin [4], expresses that "*[...] the sum of the group-delays in the two directions, which may be regarded as the 'round-trip delay', is equal to the sum of the energies stored per watt input when the network is excited from each port.*" In the case when the two-port is reciprocal and symmetrical (or antimetrical), the relationship given in equation (29) in the above paper can be deduced directly from this equation. We were unaware of the work in [1]–[4] at the time and we would like to apologize for claiming credit for this result.

REFERENCES

[1] P. P. Penfield, R. Spence, and S. Duinker, *Tellegen's Theorem and Electrical Networks*, ser. Monograph. 58. Cambridge, MA: MIT Press, 1970.

[2] R. H. Dicke, *Principles of Microwave Circuits*, ser. MIT Rad. Lab.. New York: McGraw-Hill, 1948, ch. 5.

[3] G. Kishi and K. Nakazawa, "Relations between reactive energy and group delay in lumped-constant networks," *IEEE Trans. Circuit Theory*, vol. CT-10, pp. 67–71, Mar. 1963.

[4] H. J. Carlin, "Network theory without circuit elements," *Proc. IEEE*, vol. 55, pp. 482–497, Apr. 1967.

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[1]C. Ernst, V. Postoyalko, and N. G. Khan, *IEEE Trans. Microwave Theory Tech.*, vol. 49, no. 1, pp. 192–196, Jan. 2001.